

# Backbone Construction for Heterogeneous Wireless Ad Hoc Networks

Hui Guo<sup>†</sup>, Yi Qian<sup>†</sup>, Kejie Lu<sup>‡</sup> and Nader Moayeri<sup>†</sup>

<sup>†</sup>National Institute of Standards and Technology  
Gaithersburg, Maryland 20899, USA

<sup>‡</sup>University of Puerto Rico  
Mayaguez, Puerto Rico 00681, USA

**Abstract**—In this paper, we propose a backbone construction scheme over heterogeneous ad hoc networks, where the network nodes have different characteristics such as communication capacity, processing power and energy resource. Most of the wireless backbone construction techniques focus on minimizing the number of backbone nodes. In our proposed scheme, we not only minimize the backbone size, but also take the characteristics of nodes into account when building a backbone. In the scheme, the more capable nodes have higher probability to serve as backbone nodes and provide a wireless highway over which end-to-end communication can take place. The proposed scheme includes two major steps, which can be solved by formulating as a Dominating Set (DS) problem and a Steiner Tree Problem with Minimum Number of Steiner Points (STP-MSP) respectively. We focus on the two subproblems and present a number of polynomial time approximation algorithms. Simulation results show that the proposed scheme achieves higher average backbone node performance, while has approximately the same backbone size comparing with other schemes.

## I. INTRODUCTION

Hierarchical techniques [1][2] have been studied to provide scalability for ad hoc networks with smaller routing tables and reduced routing overhead. Backbone structure [3][4] is one of the typical techniques for hierarchical infrastructure. However, the objective of most of prior work for backbone construction is to minimize the number of backbone nodes, where the characteristics of network nodes have not been taken into account. Recently, many ad hoc network applications are based on heterogeneous networks. Such a network is composed of different wireless devices with different communication capacities, processing and energy resources, or interfaces (e.g., 802.11a/b/g, WiMax, UWB). For example, for an ad hoc network in battlefield environment, network nodes have a wide range of capabilities. Those can include high altitude aircraft or satellites, low flying unmanned aerial vehicles (UAVs), ground vehicles, soldiers, and sensors. Some of these nodes, such as satellites or UAVs may have robust and powerful communication capacities, while others such as soldiers or sensors may have limited and time varying communication capacities with limited transmission power. In a public wireless network, also, the network could be composed of some dedicated wireless routers, desktop machines, laptops, handhelds, and phones, where some nodes are more powerful and others are less powerful for communications and processing.

In most of the current backbone construction schemes, all devices are considered equal when performing backbone nodes (BNs) selection. That is, the weak nodes have the same

probability to be selected as BNs as strong nodes. In our scheme, we take the characteristics of nodes into account when constructing a backbone, where the characteristics could be communication capacity, processing power, energy resource, etc., or any combination of them. We term the characteristics as *capability* in this paper. The more capable nodes have higher priority to be selected as BNs. There are two major steps in our backbone construction scheme. The first step is to select a *dominating set* [5] of the nodes, which means all nodes in the graph are either in the dominating set, or adjacent (one-hop) to at least one of the nodes in that set. In order to select the more capable nodes to serve as dominators (i.e., BNs), a capability level is specified for each nodes. We select the nodes with highest capability level as dominators until all nodes are covered (i.e., one-hop away from one of dominators). The second step is to put some relay nodes into the network to provide the connectivity of selected BNs. The problem of minimizing the number of relay nodes (Steiner points) is termed as STP-MSP (Steiner tree problem with minimum number of Steiner points) [6]. In practical applications, the relay nodes could be devices combining with communication and mobility capabilities (e.g. mobile robots or throwboxes [7]), or could be ground or aerial vehicles which provide the required mobility for wireless devices. We assume that the relay nodes have at least the same capability as selected BNs. In this way, the capable nodes in the network are connected through the deployment of relay nodes to form into a wireless highway over which reliable and high bandwidth end-to-end communications can take place.

The problem of constructing a backbone structure over wireless ad hoc networks has been widely investigated. A great amount of existing research devoted to backbone formation based on the *minimum connected dominating set* (MCDS) concept [3][4][8][9], which means, the selected nodes in dominating set must together induce a connected graph, and the number of selected nodes is minimized [10]. However, the *connected dominating set* scheme is only suitable for well-connected networks, i.e., an end-to-end path is guaranteed to exist between any source-destination pair. Our proposed backbone construction scheme is based on DS-based backbone nodes plus relay nodes placement, which is applicable to different kinds of network scenarios, and we have proved that our scheme would result in smaller backbone size than MCDS approaches.

The major contributions of this paper include the following:

1) Our backbone construction scheme is based on finding the most capable nodes to serve as BNs, while most of previous work have the sole objective of minimizing the backbone size; 2) We have proved that our constructed backbone achieves higher average BN performance than other approaches through simulations; 3) A new algorithm for the problem of BN selection is presented and analyzed; 4) For the relay node placement problem, we present a polynomial-time approximation algorithm with a smaller approximation ratio than previous work; 5) This paper is the first work to decompose backbone formation problem into two subproblems of dominating set and STP-MSP to solve, which provides a unified backbone construction approach for different network scenarios (i.e., dense or sparse, connected or partitioned network scenarios) while the others are under the assumption that the network is dense and well-connected.

The rest of this paper is organized as follows. In Section II, we present the notation and problem formulation. Section III presents and analyzes our backbone construction algorithms. Simulation results are presented in Section IV. Finally, we conclude the paper in Section V.

## II. BACKGROUND AND PROBLEM FORMULATION

### A. Notation

We use the following notation in the rest of the paper. For more graph theoretic terms that are not defined in this paper, readers can refer to the text book [5]. For two points  $x$  and  $y$  in the *Euclidean* plane, we use  $[x, y]$  to denote the *line segment* connecting them and  $\|xy\|$  to denote the *Euclidean distance* between them. We denote  $V$  as the collection of nodes  $\{v_1, v_2, \dots, v_n\}$ , and  $\mathcal{X}$  as the collection of BNs  $\{v_{x_1}, v_{x_2}, \dots, v_{x_k}\}$  selected from  $V$ , where  $\{x_1, x_2, \dots, x_k\} \subseteq \{1, 2, \dots, n\}$ ,  $k \leq n$ . We define a node  $v_i$  as a regular node (RN) when  $v_i \in V \setminus \mathcal{X}$ . We assume the location of all nodes can be detected through GPS or be obtained through a localization mechanism. The location of node  $v_i$  is denoted by  $x$ - $y$  tuples  $(x_{v_i}, y_{v_i})$ . We denote  $\mathcal{Y}$  as the collection of relay nodes  $\{y_1, y_2, \dots, y_l\}$ , and  $\mathcal{B}$  as the overall collection of BNs  $\{b_1, b_2, \dots, b_{k+l}\}$ , where  $\mathcal{B} = \mathcal{X} \cup \mathcal{Y}$ . We assume a *disk* connectivity model for the communication channel. That is, each node is equipped with an omnidirectional antenna that makes the transmission coverage a disk, whereby two nodes can communicate if and only if they are within a certain communication range. We assume that each node ( $v_i$ ) has corresponding values  $C_i$  and  $r_i$  that indicate the capability level and radio range of that node, respectively. Let  $\mathcal{R} = \{r_1, r_2, \dots, r_n\}$ . In our analysis we assume the relay nodes have the same radio range with  $R = \max_{r_i \in \mathcal{R}}(r_i)$ , so  $R \geq r_i > 0$ . The network topology can be represented as an undirected graph  $G = (V, E)$  where  $V$  is a set of vertices including all nodes and  $E$  is a set of undirected edges. An edge between nodes  $v_i$  and  $v_j$  indicates their distance is at most the radio range of  $r_i$  and  $r_j$ , i.e.,  $\|v_i v_j\| \leq \min(r_i, r_j)$ . Nodes  $v_i, v_j \in V$  are said to be neighbors if  $(v_i, v_j) \in E$ .  $N(v)$  denotes the set of neighbors of node  $v$  and  $d_G(v)$  denotes the degree of

node  $v$  in graph  $G$ . Suppose the maximum node degree of all nodes is  $\Delta$ , i.e.,  $\Delta = \max_{v \in V}(d_G(v))$ . In the dominating set problem, the set of nodes that are associated with (covered by) a dominator  $v_j \in \mathcal{X}$  is denoted by  $D(v_j)$ , and we have  $D(v_j) = N(v_j) \setminus \mathcal{X}$ . We assume that a heterogeneous ad hoc network has  $1, 2, \dots, L_{max}$  capabilities levels, with  $L_{max}$  the highest capability level among all the nodes.

### B. Problem Formulation

In the rest of this paper, the problem of constructing a minimum wireless backbone for an ad hoc network with high node capability level is termed as MWBHNC, the problem of finding a minimum dominating set of nodes is termed as MDS, the problem of finding a minimum dominating set of nodes with high node capability level (the first step) is termed as MDSHNC, and the problem of minimum number of relay nodes placement (the second step) is termed as MRNP.

*Definition 3.1:* Let  $V = \{v_1, v_2, \dots, v_n\}$  be a set of nodes in network and  $\mathcal{R} = \{r_1, r_2, \dots, r_n\}$  be their corresponding radio ranges,  $\mathcal{X}$  be a set of BNs selected from  $V$ ,  $\mathcal{Y}$  be a set of relay nodes and  $R$  be their radio ranges. The *hybrid communication graph*  $G(\mathcal{R}, R, M, E_{rR})$  induced by the 4-tuple  $(\mathcal{R}, R, M, E_{rR})$  is an undirected graph with vertex set  $M = V \cup \mathcal{Y}$  and edge set  $E_{rR}$ .  $E_{rR}$  is defined as follows:

- For any two nodes  $v_i, v_j \in V$ , if  $\|v_i v_j\| \leq \min(r_i, r_j)$ , then the edge  $(v_i, v_j) \in E_{rR}$ ;
- For any node  $v_i \in V$  and any relay node  $y_j \in \mathcal{Y}$ , if  $\|v_i y_j\| \leq r_i$ , then the edge  $(v_i, y_j) \in E_{rR}$ ;
- For any two relay nodes  $y_i, y_j \in \mathcal{Y}$ , if  $\|y_i y_j\| \leq R$ , then the edge  $(y_i, y_j) \in E_{rR}$ .

**Problem MWBHNC:** Given  $\mathcal{R}$ ,  $R$ , and the graph  $G(V, E)$  where  $V$  denotes the set of nodes and  $E$  denotes the set of edges such that  $E = \{(v_i, v_j) | v_i, v_j \in V, \|v_i v_j\| \leq \min(r_i, r_j)\}$ . The MWBHNC problem is to find a minimum set of backbone nodes (i.e.,  $\min(|\mathcal{B}|)$ ,  $\mathcal{B} = \mathcal{X} \cup \mathcal{Y}$ ) and a hybrid communication graph  $G(\mathcal{R}, R, M, E_{rR})$ , where  $M = V \cup \mathcal{Y}$ . The backbone is a *tree*  $T(\mathcal{R}, R, M, E_T)$  ( $E_T \subseteq E_{rR}$ ) over  $G(\mathcal{R}, R, M, E_{rR})$  such that:

- For any edge  $e_i \in E_T$  connecting two nodes between  $v_i$  and  $v_j$ , if  $v_i \in V \setminus \mathcal{X}$  and  $v_j \in \mathcal{X}$ , we have  $\|v_i v_j\| \leq \min(r_i, r_j)$ ;
- For any edge  $e_i \in E_T$  connecting two nodes between  $v_i$  and  $y_j$ , if  $v_i \in \mathcal{X}$  and  $y_j \in \mathcal{Y}$ , we have  $\|v_i y_j\| \leq r_i$ ;
- For any edge  $e_i \in E_T$  connecting two nodes between  $y_i$  and  $y_j$ , if  $y_i, y_j \in \mathcal{Y}$ , we have  $\|y_i y_j\| \leq R$ ;
- $\forall v_i \in V$ , we have either  $v_i \in \mathcal{X}$  or  $\exists v_j \in \mathcal{X}, v_i \in N(v_j)$ ;
- $\forall v_i \in V \setminus \mathcal{X}, \exists v_j \in \mathcal{X}$ , such that  $v_i \in D(v_j)$ , and  $C_j = \max_{v_k \in N(v_i)}(C_k)$  (any regular node is covered by its neighbor with highest capacity level).

**Problem MDS:** Given the graph  $G(V, E)$  where  $V$  denotes the set of nodes and  $E$  the set of edges such that  $E = \{(v_i, v_j) | v_i, v_j \in V, \|v_i v_j\| \leq \min(r_i, r_j)\}$ . The MDS problem is to find a minimum set of nodes  $\mathcal{X}$  selected from  $V$  such that:

- $\forall v_i \in V$ , we have either  $v_i \in \mathcal{X}$  or  $\exists v_j \in \mathcal{X}, v_i \in N(v_j)$ .

**Problem MDSHNC:** Given the graph  $G(V, E)$  where  $V$  and  $E$  denote the sets of nodes and edges such that  $E = \{(v_i, v_j) | v_i, v_j \in V, ||v_i v_j|| \leq \min(r_i, r_j)\}$ . The MDSHNC problem is to find a minimum set of nodes (i.e.,  $\min(\mathcal{X})$ ) with highest node capability selected from  $V$  such that:

- $\forall v_i \in V$ , we have either  $v_i \in \mathcal{X}$  or  $\exists v_j \in \mathcal{X}, v_i \in N(v_j)$ ;
- $\forall v_i \in V \setminus \mathcal{X}, \exists v_j \in \mathcal{X}$ , such that  $v_i \in D(v_j)$ , and  $C_j = \max_{v_k \in N(v_i)} (C_k)$ .

**Problem MRNP:** Given  $\mathcal{R}, R$  and the graph  $G(\mathcal{X}, E)$  where  $\mathcal{X}$  denotes the set of selected backbone nodes and  $E$  denotes the set of edges such that  $E = \{(v_i, v_j) | v_i, v_j \in \mathcal{X}, ||v_i v_j|| \leq \min(r_i, r_j)\}$ . The MRNP problem is to find a minimum set of relay nodes (i.e.,  $\min(\mathcal{Y})$ ) and a Steiner tree  $T(\mathcal{R}, R, \mathcal{X} \cup \mathcal{Y}, E_T)$  such that:

- For any edge  $e_i \in E_T$  connecting two nodes between  $v_i$  and  $v_j$ , if  $v_i, v_j \in \mathcal{X}$ , we have  $||v_i v_j|| \leq \min(r_i, r_j)$ ;
- For any edge  $e_i \in E_T$  connecting two nodes between  $v_i$  and  $y_j$ , if  $v_i \in \mathcal{X}$  and  $y_j \in \mathcal{Y}$ , we have  $||v_i y_j|| \leq r_i$ ;
- For any edge  $e_i \in E_T$  connecting two nodes between  $y_i$  and  $y_j$ , if  $y_i, y_j \in \mathcal{Y}$ , we have  $||y_i y_j|| \leq R$ .

The main purpose of the problems above is to minimize backbone size while the selected BNs have higher capability than others. Both MDSHNC and MRNP problems are *NP-hard*. In this paper, we propose some faster polynomial-time approximation algorithms to solve these problems. A *polynomial time  $\alpha$ -approximation algorithm* for a minimization problem is defined as an algorithm  $\mathcal{A}$  such that, for any instance of the problem, computes a solution that is at most  $\alpha$  times of the optimal solution of the instance, in time bounded by a polynomial in the input size of the instance [11]. In this case, we also say that  $\mathcal{A}$  has an approximation ratio of  $\alpha$ .

To the best of our knowledge, this paper is the first attempt to deal with the MWBHNC problem. We decompose the problem into two subproblems, MDSHNC and MRNP, to solve separately in this paper. The MDSHNC problem have not been investigated before although its general case - MDS problem has been widely studied. An algorithm with approximation ratio of  $H(\Delta + 1)$  is presented for MDSHNC problem, where  $H$  is the harmonic function. The MRNP problem has been studied in [12], where the authors have proposed a simple minimum spanning tree (MST) based approximation algorithm for MRNP with approximation ratio of 7. In this paper we present an  $O(n^3)$  algorithm with a smaller approximation ratio of 3.

### III. THE BACKBONE CONSTRUCTION ALGORITHMS

#### A. Finding Dominators

The objective of MDSHNC is to find a minimum set of dominators that the capability of each dominator is the largest one among the neighbors of the nodes associate with (covered by) that dominator. We first investigate its general case, MDS problem, then modify the general case to solve the MDSHNC problem. It is known that MDS problem is equivalent to *minimum set cover* (MSC) problem [5]. A set cover instance is created by making each vertex an element, and each vertex  $v_i$

corresponds to a set  $S_i$  that contains the vertex itself, together with its neighbors, i.e.,  $S_i = \{v_i\} \cup \{v_j | v_j \in N(v_i)\}$ .

*Definition 4.1:* Given a finite set  $\mathcal{U}$  and a family of  $n$  non-empty subsets  $\Omega = S_1, S_2, \dots, S_n$  that their *universe* is  $\mathcal{U}$  (i.e.,  $S_1 \cup S_2 \cup \dots \cup S_n = \mathcal{U}$ ), the **MSC Problem** is to find a minimum cardinality  $X \subseteq \{1, 2, \dots, n\}$  such that  $\bigcup_{i \in X} S_i = \mathcal{U}$ .

*Definition 4.2:* Given  $X \subseteq \{1, 2, \dots, n\}$ , an element is said to be *covered* if it belongs to  $\bigcup_{i \in X} S_i$ .

It is known that the MSC problem is *NP-hard*. Here we present a greedy heuristic algorithm for MDS problem. As shown in *Algorithm 1*, the idea is that at each iteration, we always pick the set containing the maximum number of uncovered elements, until all elements are covered. For the sets that are selected for covering the *universe*  $\mathcal{U}$ , their corresponding vertices,  $v_i$  ( $i \in X$ ), are selected dominators.

---

#### Algorithm 1: Greedy Algorithm for MDS problem

---

**input** : Family  $\Omega = S_1, S_2, \dots, S_n$  of subsets of a finite set  $\mathcal{U}$ ;  $\{C_1, C_2, \dots, C_n\}$ ;

**output**:  $\mathcal{X} \subseteq V$

- 1  $\mathcal{F} := \mathcal{U}$ ; ( $\mathcal{F}$  is the set of vertices that are currently uncovered)
- 2  $\mathcal{X} := \emptyset$ ;
- 3  $G := \{1, 2, \dots, n\}$ ;
- 4 **while**  $\mathcal{F} \neq \emptyset$  **do**
- 5     choose a subset  $S_i$  where  $i \in G$  such that  $|S_i \cap \mathcal{F}|$  is maximum among all  $\{S_j | j \in G\}$ ;
- 6      $\mathcal{F} := \mathcal{F} \setminus S_i$ ;
- 7      $\mathcal{X} := \mathcal{X} \cup \{v_i\}$ ;
- 8      $G := G \setminus \{i\}$ ;
- 9 **end**
- 10 output  $\mathcal{X}$ ;

---

It is known [13] that a greedy algorithm is an  $H(k)$ -approximation algorithm for minimum set cover problem, where  $k$  denotes that each  $S_i$  has at most  $k$  elements (i.e.,  $|S_i| \leq k$  for all  $S_i \in \Omega$ ). Since  $k = \Delta + 1$ , we know that the *Algorithm 1* is an  $H(\Delta + 1)$ -approximation algorithm for MDS problem.

The objective of the MDSHNC problem can be achieved by modifying *Algorithm 1*. As shown in *Algorithm 2*, the idea is that at each iteration, we pick the node from the set of  $L_{max}$  level nodes that can cover maximum number of uncovered nodes. After the selection of the  $L_{max}$  level nodes as possible dominators, if there still exists uncovered nodes, then we select the  $(L_{max} - 1)$  level nodes as possible dominators. This process will continue until all nodes in the network are covered.

*Theorem 4.1:* Using the greedy algorithm described in *Algorithm 2* yields a dominating set of size at most  $H(\Delta + 1)|OPT|$ , where  $OPT$  is the minimum dominating set for DSHNC problem,  $\Delta$  is the maximum node degree.

**PROOF:** Since the greedy algorithm is a  $H(\Delta + 1)$ -approximation algorithm for MDS problem, when  $L_{max} = 1$ , the MDSHNC problem turn into MDS problem, we have the

**Algorithm 2:** Greedy Algorithm for MDSHNC

---

**input** : Family  $\Omega = S_1, S_2, \dots, S_n$  of subsets of a finite set  $\mathcal{U}$ ;  $\{C_1, C_2, \dots, C_n\}$ ;  
**output**:  $\mathcal{X} \subseteq V$

- 1  $\mathcal{F} := \mathcal{U}$ ;
- 2  $\mathcal{X} := \emptyset$ ;
- 3  $L_{max} := \max_{i \in \{1, 2, \dots, n\}} (C_i)$ ;
- 4 **for**  $level := 1$  **to**  $L_{max}$  **do**
- 5    $G^{(level)} := \emptyset$ ;
- 6 **end**
- 7 **for**  $i := 1$  **to**  $n$  **do**
- 8    $G^{(C_i)} := G^{(C_i)} \cup \{i\}$ ;
- 9 **end**
- 10  $level := L_{max}$ ;
- 11 **while**  $\mathcal{F} \neq \emptyset$  **do**
- 12   choose a subset  $S_i$  where  $i \in G^{(level)}$  such that  $|S_i \cap \mathcal{F}|$  is maximum among all  $\{S_j | j \in G^{(level)}\}$ ;
- 13    $\mathcal{F} := \mathcal{F} \setminus S_i$ ;
- 14    $\mathcal{X} := \mathcal{X} \cup \{v_i\}$ ;
- 15    $G^{(level)} := G^{(level)} \setminus \{i\}$ ;
- 16   **if**  $\forall i \in G^{(level)}, |S_i \cap \mathcal{F}| = 0$  **then**
- 17      $level := level - 1$ ;
- 18   **end**
- 19 **end**
- 20 **output**  $\mathcal{X}$ ;

---

approximation ratio  $H(\Delta + 1)$  in this case. When  $L_{max} > 1$ , let  $OPT_i$  be the optimal solution for MDS selected from the set of nodes with capability level  $i$ ,  $\mathcal{B}_i$  be the dominators selected from the set of nodes with capability level  $i$  through *Algorithm 2*, then we have  $|\mathcal{B}_i| \leq H(\Delta + 1)|OPT_i|$ . Hence, the total number of dominators through *Algorithm 2*,  $\sum_{i=1}^{L_{max}} |\mathcal{B}_i|$ , is no more than  $H(\Delta + 1) \sum_{i=1}^{L_{max}} |OPT_i|$ . Since  $|OPT| = \sum_{i=1}^{L_{max}} |OPT_i|$ , we have  $\sum_{i=1}^{L_{max}} |\mathcal{B}_i| \leq H(\Delta + 1)|OPT|$ .  $\square$

**B. Relay Node Placement**

In this subsection we focus on the second subproblem that if a set of dominators ( $\mathcal{X}$ ) is given, we need to deploy the minimum number of relay nodes in the graph  $G(\mathcal{X}, E)$  such that the resulting graph is connected. When  $R = r_i$  (i.e., all the nodes in the network and the relay nodes have the same radio range), the MRNP problem is equivalent to the standard STP-MSP problem [6]. The definition of STP-MSP is: given a set of terminals  $\mathcal{X} = \{x_1, x_2, \dots, x_n\}$  and a positive constant  $r$ , find a tree  $T$  spanning  $\mathcal{X}$  such that each edge in the tree has a length no more than  $r$  and the number of Steiner points (nodes other than those in  $\mathcal{X}$ ), is minimized. In [12] the authors have studied the MRNP problem and an 7-approximation algorithm was proposed. Combining with the ideas in [14] which target for STP-MSP, we present an  $O(n^3)$  approximation algorithm with approximation ratio of 3 for MRNP problem.

*Definition 4.3:* Given a set of nodes  $\mathcal{X} = \{v_1, v_2, \dots, v_k\}$  and their radio ranges  $\mathcal{R} = \{r_1, r_2, \dots, r_k\}$ . Given the radio range of relay nodes  $R$ .  $T_{mst}(\mathcal{X})$  is defined as the *minimum spanning tree* (MST) over the nodes in  $\mathcal{X}$ . The *steinerized*

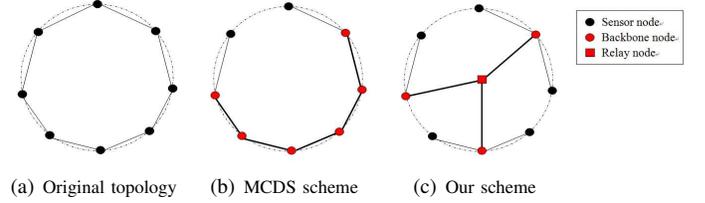


Fig. 1. An example of MCDS-based scheme and our scheme

*minimum spanning tree*  $T_{mst}^S(\mathcal{X})$  is defined by placing a minimum number of relay nodes ( $y_k \in \mathcal{Y}$ ) on the line segment  $[v_i, v_j]$  for each edge  $e = (v_i, v_j)$  in  $T_{mst}(\mathcal{X})$ , such that  $\|v_i, y_k\| \leq r_i$  for each edge  $(x_i, y_k)$ , and  $\|y_i, y_j\| \leq R$  for each edge  $(y_i, y_j)$  in  $T_{mst}^S(\mathcal{X})$ .

Most of existing work (e.g., [6][12]) are based on *steinerized minimum spanning tree* approach. Since the relay nodes are in general placed along the edges of the MST, these work cannot find solutions in which a relay node is used as a central junction that connects multiple nodes. We present a 3-star approach that for each combination of 3 disconnected nodes, exploring the possibility to deploy a relay node to connect them. With the central junction of relay nodes, we believe that our scheme would result in smaller backbone size than MCDS-based approaches. An extreme example is that all the eight nodes are equally distributed around a circle, as shown in Fig. 1. The edges in the graph denote that every two nodes connected by them are within the radio range of each other. We can see that the backbone size of MCDS-based scheme is 6, while our scheme (the sum of BNs and relay nodes) is 4.

*Lemma 4.1:* If a triangle  $ABC$  is acute, the minimum enclosing circle of the triangle is the one bounded by the circle circumscribing  $ABC$ . If a triangle  $ABC$  is obtuse or right, the minimum enclosing circle of the triangle is the one whose diameter is the longest edge of triangle  $ABC$ . (The proof is obvious and it is omitted due to the limit of space.)

For an acute triangle, its circumcenter can be found as the intersection of the three perpendicular bisectors. If the radius of MEC is no larger than the minimum radio range of A, B and C, put a relay node in circumcenter to connect the three separated dominators. We denote the process as *CircleJudge* ( $A, B, C$ ). The circumcenter is denoted as  $O$ .

*Definition 4.5:* A *connected cluster* in ad hoc networks is defined as for any two nodes in the same connected cluster, there is an end-to-end path between them. For any two nodes located in different connected cluster, there is no end-to-end path between them.

As shown in *Algorithm 3*, we try to find a 3-star for any three nodes located in different connected cluster. Then, we steinerize the line segments  $[v_i, v_j]$  with  $\|v_i, v_j\|$  in increasing order where  $v_i, v_j$  are located in different connected clusters. The operation *Combining*( $v_i, v_j$ ) combines two clusters containing  $v_i$  and  $v_j$  respectively into an integrated one.

*Theorem 4.3:* Let  $\mathcal{Y}$  be the set of relay nodes obtained from *Algorithm 3*, we have  $|\mathcal{Y}| \leq 3 \cdot |OPT|$ . (See *Appendix* for proof).

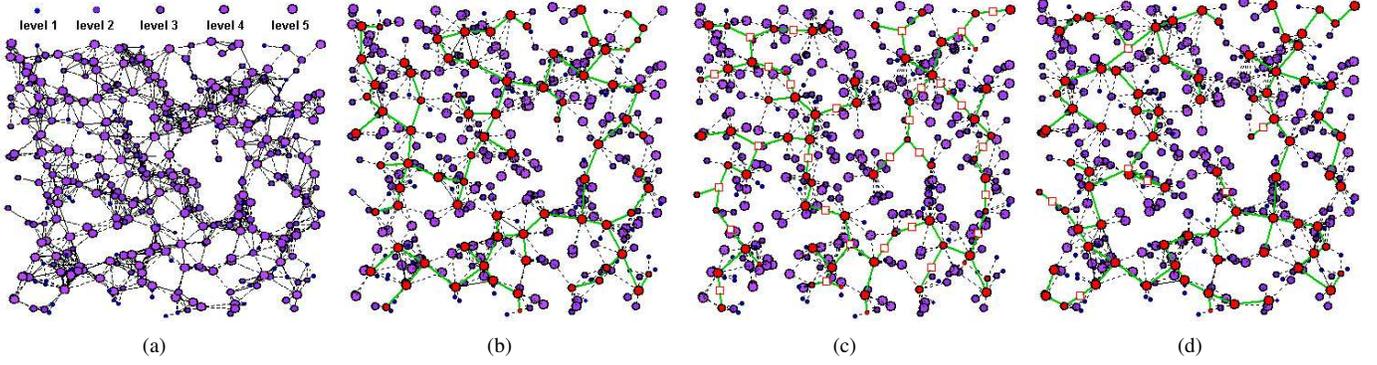


Fig. 2. a) Original topology; b) MCDS scheme backbone; c) MDS+MRNP scheme backbone; d) MDSHBC+MRNP scheme backbone.

---

**Algorithm 3:** Approximation algorithm for MRNP

---

**input :** A set of dominators  $\mathcal{X} = \{v_1, v_2, \dots, v_k\}$ ,  $\mathcal{R}$ ,  $R$   
**output:** A Steiner tree  $T_s(\mathcal{X})$

```

1  $T_s(\mathcal{X}) = \emptyset$ ;
2 for each line segment  $[v_i, v_j]$  where  $x_i, x_j$  located in
   different connected clusters do
3   if  $\|v_i v_j\| \leq \min(r_i, r_j)$  then
4      $\text{Combining}(v_i, v_j)$ ; put  $[v_i, v_j]$  into  $T_s(\mathcal{X})$ ;
5   end
6 end
7  $T^{(1)} := T_s(\mathcal{X})$ ;
8 for each set of three points  $A, B, C$  located in three
   different connected clusters do
9   if  $\text{CircleJudge}(A, B, C) = \text{success}$  then
10     $\text{Combining}(A, B, C)$ ;
11    put  $[O, A], [O, B], [O, C]$  into  $T_s(\mathcal{X})$ ;
12  end
13 end
14  $T^{(2)} := T_s(\mathcal{X})$ ;
15 for each  $[v_i, v_j]$  with  $\|v_i v_j\|$  in the increasing order do
16   if  $v_i, v_j$  located in different connected clusters then
17     put steinerized  $[v_i, v_j]$  into  $T_s(\mathcal{X})$ ;
18      $\text{Combining}(v_i, v_j)$ ;
19   end
20 end

```

---

#### IV. PERFORMANCE EVALUATION

Three different backbone construction schemes, MCDS, MDS+MRNP and MDSHNC+MRNP are implemented in MATLAB to evaluate the performance and to compare our scheme (i.e., MDSHNC+MRNP) with other two schemes (i.e., MCDS and MDS+MRNP). In the simulations, a total of  $n$  nodes are randomly distributed in a region of  $2500 m \times 2500 m$ . We assume the nodes with higher capability level have larger radio ranges, and the ranges are between  $150 m$  and  $350 m$ , and all relay nodes have fixed radio range of  $350 m$ . Fig. 2 gives an example of the original deployment topology (Fig.2-(a)) and different backbones constructed by three different methods. In Fig. 2, the larger circles are the nodes with higher capability level (the maximum capability level is 5), the red

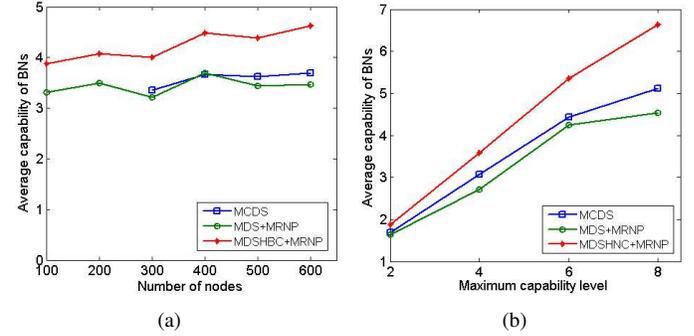


Fig. 3. a) Average capability as a function of number of nodes; b) Average capability as a function of maximum capability level.

circles are selected dominators, the squares are relay nodes and the solid green lines are backbones for the network.

In order to compare the constructed backbone performance, Fig. 3 presents the average capability (which is the average of the capability levels of the nodes that have been selected as BNs) with MCDS, MDS+MRNP and MDSHNC+MRNP schemes respectively. In Fig. 3-(a), the maximum capability level is 5, the number of nodes is ranging from 100 to 600. In Fig. 3-(b), the number of nodes is 350, the maximum capability level is ranging from 2 to 8. The results demonstrate that our scheme achieves higher average capability level than the others at all conditions, and the gap between our scheme and the others increases with the increase of maximum capability level. The backbone size (the number of dominators plus the number of relay nodes) induced by the three schemes are presented in Fig. 4. We can see that although the MDS can achieve much smaller number of dominators than MDSHNC (denoted by dashed lines), our scheme achieves approximately the same backbone size as MDS+MRNP. In addition, the MCDS scheme induces the largest backbone size among the three schemes.

#### V. CONCLUSIONS

In this paper, we have proposed a new solution to construct a backbone for heterogeneous wireless ad hoc networks. In our scheme, the nodes with higher capability (e.g., communication capacity, processing power or energy resource, etc.) have higher possibility to be selected as backbone nodes. To achieve this, a greedy algorithm for MDSHNC problem is

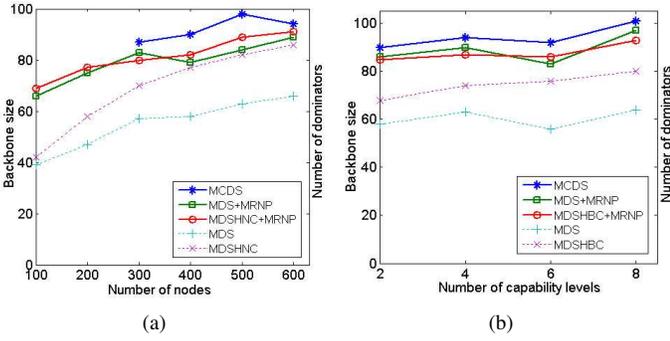


Fig. 4. a) Backbone size (or number of dominators) as a function of number of nodes; b) Backbone size (or number of dominators) as a function of maximum capability level.

proposed and analyzed. Next, we deploy a minimum number of high-performance relay nodes in the network to connect the selected backbone nodes to form into a wireless backbone over which end-to-end communication can take place. We present an approximation algorithm for MRNP problem and prove that the algorithm has smaller approximation ratio than those of previous work. Simulation results demonstrate that our backbone construction scheme achieves higher backbone performance while not increasing the backbone size.

## VI. APPENDIX

*Definition 6.1:* A Steiner tree is *full* if every terminal (i.e.,  $\forall v_i, v_i \in \mathcal{X}$ ) in the tree is a leaf. If a Steiner tree is not full, then we can always find a terminal with degree more than one which enable us to break the tree at this terminal. In this way, every Steiner tree can be broken into several smaller full Steiner trees. Those smaller full Steiner trees are called *full components* of a Steiner tree.

*Lemma 6.1:*[14] Let  $T_{OPT}$  be optimal Steiner tree on graph  $G(\chi, E)$ . Let  $T_j$  be a full component of  $T_{OPT}$ . Then the following results hold:

- (1) Every vertex in  $T_{OPT}$  has degree at most five;
- (2) The steinerized minimum spanning tree on terminals in  $T_j$  has at most  $3 \cdot |T_j| + 1$  Steiner points.
- (3) If  $T_j$  contains a Steiner point of degree at most four, then the steinerized minimum spanning tree on terminals in  $T_j$  has at most  $3 \cdot |T_j|$  Steiner points.
- (4) If the steinerized minimum spanning tree on terminals in  $T_j$  contains an edge (of length at most  $r$ ) between two terminals, then it has at most  $3 \cdot |T_j|$  Steiner points.

*Proof of Theorem 4.3:* Let  $T^{(1)}$  and  $T^{(2)}$  be temporary trees that are created by lines 7 and 14 in *Algorithm 3* respectively. Let  $T_s$  be a steinerized minimum spanning tree [6] on all given terminals ( $\mathcal{X}$ ), and assume there are  $k$  number of 3-stars have been found in  $T^{(2)}$ , then we have  $C(T_A) \leq C(T_s) - k$ . By Lemma 6.1 we know that each Steiner point of  $T_{OPT}$  has degree at most five. Suppose  $T_{OPT}$  has  $p$  number of full components. We construct a steinerized spanning tree  $T$  for  $\chi$  as follows: Initially, put  $T^{(1)}$  into  $T$ . For each full component  $T_j$  ( $1 \leq j \leq p$ ), add to  $T$  the steinerized minimum spanning tree  $T_j^s$  for terminals in  $T_j$ , i.e.,  $T = T^{(1)} \cup T_1^s \cup \dots \cup T_p^s$ . If  $T$  has a cycle, destroy the cycle by deleting some edges and Steiner points of  $T_j^s$ . An important observation is that

if  $T_j^s \cup T^{(1)}$  has a cycle, a Steiner point must be removed for destroying the cycle unless  $T_j^s$  contains an edge between two terminals. Suppose there are  $q$  number of full components ( $T_h, 1 \leq h \leq q$ ) that every Steiner point in  $T_h$  has degree five and  $T_h \cup T^{(1)}$  has no cycle, based on Lemma 6.1, we have  $C(T_s) \leq C(T) \leq 3 \cdot C(T_{OPT}) + q$ , i.e.,  $C(T_A) \leq 3 \cdot C(T_{OPT}) + q - k$ . Suppose  $T^{(1)}$  has  $m$  connected clusters, then  $T^{(2)}$  has  $m - 2k$  connected clusters  $C_i$  ( $1 \leq i \leq m - 2k$ ). We construct another graph  $G'$  with vertex set  $\chi$  and the following edges. First, put all edges of  $T^{(1)}$  into  $G'$  (which has  $m$  number of connected clusters). Then consider every full component of  $T_h$  ( $1 \leq h \leq q$ ). If  $T_h$  has only one Steiner point, then this Steiner point connects five terminals. As the five terminals could be connected by a single Steiner point, *Algorithm 3* should find a 3-star among the five terminals (in line 8). Put the edges of 3-star into  $G'$ , the number of separated terminals (clusters) in  $T_h$  are decreased from 5 to 3. If  $T_h$  has at least two Steiner points, there must exist at least two Steiner points each connecting to four terminals. Similarly, *Algorithm 3* should find a 3-star among the four terminals. Put the edges of 3-star into  $G'$ , the number of separated terminals (clusters) in  $T_h$  are decreased more than 2. Do the same operations for each  $T_h$  ( $1 \leq h \leq q$ ) on  $G'$ . After that,  $G'$  has at most  $m - 2q$  connected clusters. Since every connected cluster of  $G'$  is contained by a  $C_i$  in  $T^{(2)}$ , we have  $m - 2k \leq m - 2q \Rightarrow q \leq k$ . Therefore,  $C(T_A) \leq 3 \cdot C(T_{OPT})$ .  $\square$

## REFERENCES

- [1] J. Westcott and G. Lauer, "Hierarchical routing for very large networks," In *IEEE MILCOM*, 1984.
- [2] A. Iwata, C.-C. Chiang, G. Pei, M. Gerla, and T.-W. Chen, "Scalable routing strategies for ad hoc wireless networks," *IEEE J. Selected Areas in Comm.*, pp. 1369-1379, Aug. 1999.
- [3] B. Das, R. Sivakumar, and V. Bharghavan, "Routing in ad-hoc networks using a virtual backbone," In *IEEE ICCCN*, 1997.
- [4] X. Cheng, M. Ding, H. Du and X. Jia, "Virtual backbone construction in multihop ad hoc wireless networks," *Wireless Communications and Mobile Computing*, 6: 183-190, 2006.
- [5] D. B. West, *Introduction to Graph Theory*, Prentice Hall, 1996.
- [6] G. Lin and G. Xue, "Steiner tree problem with minimum number of Steiner points and bounded edge-length," *Information Processing Letters*, 69(2): 53-57, 1999.
- [7] N. Banerjee, M. D. Corner, and B. N. Levine, "An energy-efficient architecture for DTN throwboxes," In *IEEE INFOCOM*, 2007.
- [8] K. M. Alzoubi, P. J. Wan, and O. Frieder, "New distributed algorithm for connected dominating set in wireless ad hoc networks," *Proc. IEEE Hawaii Int'l Conf. System Sciences*, 2002.
- [9] F. Dai and J. Wu, "An extended localized algorithm for connected dominating set formation in ad hoc wireless networks," *IEEE Transactions on Parallel and Distributed Systems*, vol. 15, pp. 908-920, 2004.
- [10] S. Guha and S. Khuller, "Approximation algorithms for connected dominating sets," *Algorithmica*, 20(4):374-387, 1998.
- [11] T. Cormen, C. Leiserson, R. Rivest, and C. Stein, *Introduction to Algorithms*, MIT Press and McGraw-Hill, 2001.
- [12] E. L. Lloyd and G. Xue, "Relay node placement in wireless sensor networks," *IEEE Transactions on Computers*, 56(1): 134-138, 2007.
- [13] V. Chvatal, "A greedy heuristic for the set covering problem," *Mathematics of Operations Research*, 4:233-235, 1979.
- [14] D. Chen, D. Du, X. Hu, G. Lin, L. Wang, and G. Xue, "Approximations for steiner trees with minimum number of steiner points," *Journal of Global Optimization*, 18:17-33, 2000.